Book of Abstracts

4th Interdisciplinary Scientific Conference

Mathematical Transgressions

13 – 16 March 2019

Pedagogical University of Cracow

Poland

ISCMT 2019 Program

Wednesday, 13th March 2019

8.30 – 9.00 Registration (*in front of Auditorium*)

9.00 – 9.15 Opening Ceremony (*Auditorium*)

9.15 – 10.15 David Tall, University of Warwick, UK

Complementing supportive and problematic aspects of mathematics to resolve transgressions in long-term sense making

10.15 – 10.45 Coffee break *(Aula)*

10.45 – 11.45 Ingo Witzke, University of Siegen, Germany

Epistemological Beliefs about Mathematics in Education

11.45 – 12.00 Moving break

12.00 – 13.00 Zbigniew Semadeni, University of Euroregional Economy, Józefów-Warsaw, Poland

Rhythms, gestures, subitizing and learning to count

13.00 – 14.30 Lunch break (Aula)

14.30 – 15.30 Władimir Mitiuszew, Pedagogical University of Cracow, Poland

Higher education in computer epoch. How to teach biology, chemistry, economy, engineering, geography, informatics ... and mathematics?

15.30 – 16.30 Session 1A: New technologies and mathematics

Felicitas Pielsticker, University of Siegen, Germany

Concept Building in Mathematics Classrooms using New Media: Theoretical Perspectives and Empirical Insights on the Example of 3D-Printing Technology

Sylwia Kania, University of Silesia, Katowice, Poland

Transgressions between "old" methods of problem solving and "new" students

16.30 – 16.45 Moving break

16.45 – 17.45 Session 1B: New technologies and mathematics

Katarzyna Wadoń-Kasprzak, Witold Pilecki University of Applied Sciences in Oświęcim, Poland

Forming the concept of parameter through the work with computer algebra software

Janina Duda, Witold Pilecki University of Applied Sciences in Oświęcim, Poland

Mathematical transgressions of gifted students inspired by using information technology

20.00 – 22.30 Conference Dinner (*Pistacja Restaurant, Karmelicka 7*)

Thursday, 14th March 2019

9.00 – 10.00 Rina Zazkis, Simon Fraser University, Canada

On interplay between research, mathematics and pedagogy

10.00 – 10.30 Coffee break

10.30 – 12.00 Session 2: Mathematics teaching

Adedeji Tella, University of Ibadan, Nigeria

Use of self-regulated learning strategies by senior secondary school mathematics students

Mirosława Sajka, Pedagogical University of Cracow, Poland

The notion of function as a mathematical study of movement - an excerpt from eye-tracking research

Roberto Tortora, Università degli Studi di Napoli Federico II, Italy

Very often students' errors are all but mistakes

12.00 – 12.15 Moving break

12.15 – 13.45 Session 3: Mathematics teachers' education

Stanislav Lukáč, Tadeáš Gavala, Pavol Jozef Šafárik University in Košice, Slovakia

Development of pre-service mathematics teachers' competencies for inquiry-based teaching

Nelleke den Braber, NHL Stenden University of Applied Sciences, Netherlands

Jenneke Krüger, Freudenthal Institute, University of Utrecht, Netherlands

Gaining perspective: lessons from interviews with non-mathematics teachers by pre-service mathematics teachers

Basia Pieronkiewicz, Pedagogical University of Cracow, Poland

High-school mathematics textbooks as the potential source of pre-service teachers' images of a line tangent to a plane curve

13.45 – 15.00 Lunch break

15.00 – 16.30 Session 4: History of mathematics

Janet Barnett, Colorado State University – Pueblo, USA

Transgressions in Nineteenth-century Mathematics: Meaningful Context for Today's Classroom

András Benedek, Research Centre for the Humanities, Hungarian Academy of Sciences, Hungary

On the Role of Model Theory in Understanding Mathematical Transgressions: Some examples from the history of continuity, computability and categoricity

Marlena Fila, Pedagogical University of Cracow, Poland

On continuity in Bolzano's 1817Rein analytischer Beweis

16.30 – 17.00 Coffee break

17.00 – 18.30 Workshop (room 118)

Piotr Błaszczyk, Pedagogical University of Cracow, Poland

On Euler's formula. Between standard and non-standard analysis

Friday, 15th March 2019

9.00 – 10.00 Anna Sfard, University of Haifa, Israel

Discursive gaps in mathematics classroom: the invisible pitfalls of routines

10.00 – 10.30 Coffee break

10.30 – 13.00 Session 5: Communication and mathematics

Jorge Soto-Andrade, University of Chile, Chile

Daniela Diaz-Rojas, University of Oxford, UK

Alexandra Yañez-Aburto, University of Chile, Chile

Metaphorising and enacting as a means of transgression in the learning of mathematics

Gaya Jayakody, University of Sri Jayewardenepura, Sri Lanka

Realizations and commognitive conflicts in the discourse of 'continuous functions'

Janet Barnett, Colorado State University – Pueblo, USA

Cihan Can, Florida State University, USA

Kathleen Clark, Florida State University, USA

Learning Mathematics from Primary Sources: Meta-Discursive Rules, Exogenous Growth and Transgressive Acts

Ioannis Vandoulakis, Hellenic Open University, Greece

Proof-Events as a Fundamental Methodological Concept Integrating History of Mathematics into Intercultural Mathematics Education

Karl Heuer, Technical University of Berlin, Germany

Deniz Sarikaya, Universität Hamburg, Germany

Variations in open problem fields as a tool for mathematical education: From basics to open questions in the twinkling of an eye

13.00 – 14.30 Lunch break

14.30 – 15.30 Jakub Jernajczyk, Academy of Art and Design, Wrocław, Poland

Thinking in images – mathematical inspirations in contemporary conceptual art

15.30 – 15.45 Moving break

15.45 – 17.15 Session 6: Mathematics and Arts

Melissa Rodd, UCL Institute of Education, United Kingdom

Introducing mathematics through art, a case study: Maryam Mirzakhani's geometry for high school students

Agata Hoffmann, University of Wrocław, Poland

Zvi Hecker and his mathematical inspirations

Michaela Kaslová, Charles University, Czech Republic

Paola Vighi, Università di Parma, Italy

How young pupils perceive geometrical shapes and structure?

17.30 - 19.00 Farewell Meeting *(Aula)*

Saturday, 16th March 2019

9.00 – 10.00 Gerald Goldin, Rutgers University, USA

Beauty, Insight, Power, and Connection: The Conative Dimension of Mathematical Engagement

10.00 – 10.30 Coffee break

10.30 – 12.00 Session 7: Affect and mathematics

Bronisław Czarnocha, Hostos CC, CUNY, NYC, USA

Gerald Goldin, Rutgers University, USA

Peter Liljedahl, Simon Fraser University, Canada

Illuminating Aha! Moment through the Analysis of Relationships between Affect, Cognition and Conation

Kathleen Clark, Florida State University, USA

Cihan Can, Florida State University, USA

Primary Source Projects as Transgressive Acts: Perspectives on Affective Experiences

Monika Szczygieł, Pedagogical University of Cracow, Poland

Math anxiety of mothers, fathers, and teachers explains math anxiety and mathematical performance of early school-age children

12.00 – 12.30 Closing ceremony

12.30 – 14.00 Lunch

Keynote Lectures

Wednesday, 13th March

9.15 – 10.15

David Tall, University of Warwick

david.tall@warwick.ac.uk

Complementing supportive and problematic aspects of mathematics to resolve transgressions in long-term sense making

In the teaching and learning of mathematics, while it is important to focus on what happens at each stage of development, what matters even more is the cumulative effect of learning over the long-term. As mathematics grows in sophistication, new contexts require new ways of thinking that can act as barriers to progress. Passing through such a barrier may be called a *transgression.*

This presentation focuses on aspects of mathematics that remain consistent over several changes in context and contrasts them with others that cause conflict at any given stage. For instance, how we *speak,* and *write* mathematics reveals new insights into making long-term sense of increasingly sophisticated mathematical symbolism in arithmetic and algebra. How the eye tracks a moving object affects how we interpret the notion of variable in the calculus both visually on a number line and symbolically as a variable quantity.

Studying successive changes in mathematics and the positive and negative emotional affects leads to an overall framework for long-term development that applies both to historical evolution and to the individual development of different learners. It offers a practical approach in the classroom and a theoretical framework that brings together widely differing interpretations held by mathematicians, educators, curriculum designers, philosophers, neurophysiologists, and even politicians who currently specify the curriculum.

Keynote Lectures

Thursday, 14th March

9.00 – 10.00

Rina Zazkis

Simon Fraser University, Canada

zazkis@sfu.ca

On interplay between research, mathematics and pedagogy

Teacher educators' work juxtaposes mathematics and pedagogy, often seeking engagements for prospective teachers in which a task with a pedagogical flavor is used in an attempt to help teachers extend their understanding of underlying mathematics. Teaching mathematics undoubtedly relies on students' prior knowledge. However, a mathematics teacher, who meets students on a daily basis, has a general sense of her students' prior knowledge. This is often based on a prescribed curriculum sequence, as well as familiarity with students from a previous week or a previous lesson. For a mathematics teacher educator, whose goal is to extend prospective secondary school teachers' understanding of mathematics, the task is more taxing. This is especially evident in post-bachelor's degree secondary teacher education programs, where students come from different educational and mathematical backgrounds, often from different countries. How can a teacher educator get a "scan" of the group's knowledge of a mathematical topic in order to plan for, or adjust, subsequent instruction?

I will address this question with several illustrative examples. It will describe pedagogical tasks that unveil mathematical knowledge of prospective teachers and demonstrate how subsequent instructional choices build upon and expand this knowledge, highlighting links between tertiary and school mathematics.

Keynote Lectures

Friday, 15th March

9.00 – 10.00

Anna Sfard

University of Haifa, Israel

sfard@netvision.net.il

Discursive gaps in mathematics classroom: the invisible pitfalls of routines

Communication in mathematics classroom is challenged by numerous discursive gaps, most of them invisible, some of them necessary, and some potentially harmful.

The talk will open with the introduction of the notion of *routine* and with the claim that learning at large, and learning mathematics in particular, may be seen as the process of *routinization*. A number of empirical examples and theoretical arguments will then be brought to show that in many cases, students "incorrect" mathematical performances, rather than being due to the insufficient technical proficiency or to the lack of mastery in matching procedures to situations, may result from the learners' idiosyncratic interpretations of those situations. If so, learning may be seen as the process of overcoming discursive gaps between the student and the teacher. This claim, if taken seriously, must change the way in which we study the development of mathematical thinking.

The talk will conclude with a reflection on how to sensitize ourselves to discursive gaps in mathematics classroom, how to benefit from those that that are an inevitable part of learning, and how to cope with those that may hinder the process.

Keynote Lectures

Saturday, 16th March

9.00 – 10.00

Gerald Goldin

Rutgers University, USA

geraldgoldin@dimacs.rutgers.edu

Beauty, Insight, Power, and Connection: The Conative Dimension of Mathematical Engagement

Mathematical engagement – commitment to and involvement in mathematical activity as it occurs "in the moment" – is a complex, multidimensional, and dynamic construct. This talk explores aspects of its conative dimension. Conation encompasses the needs, desires, drives, goals, and/or meaningful purposes of individuals, and how these are (or are not) resolved.

I invite consideration of the question of how mathematics fulfils or can fulfil fundamental psychological needs in learners. What does this mean for the teaching of mathematics? What transgressions must occur to facilitate strong, productive motivating desires and accompanying active structures of engagement?

To address such questions, I outline a descriptive, empirically-based model for students' in-the-moment mathematical engagement during challenging classroom activity. If time permits, I will also discuss some findings from an ongoing, qualitative study of the motivating desires experienced by prospective mathematics teachers.

Plenary Lectures

Wednesday, 13th March

10.45 – 11.45

Ingo Witzke

University of Siegen, Germany

witzke@mathematik.uni-siegen.de

Epistemological Beliefs about Mathematics in Education

Based on evidence from theoretical and empirical research it is quite clear that beliefs play a decisive role in mathematical learning processes. Although it is still somewhat difficult to describe and measure direct effects, there seems to be no doubt that (epistemological) beliefs, belief systems – or in the German term, "Auffassungen" – of mathematics play a major role on "how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. The belief system establishes the context within which we operate [...]," (Schoenfeld, 1985, p. 45). Looking at material used in present mathematics classrooms and empirical studies with a focus on beliefs, for example, in the domain of Calculus, we can identify interesting parallels to historical cases in the late 17th early 18th century. Especially on the level of epistemic beliefs this view back into the history of mathematics provides us with valuable insights and legitimations, as I will argue in my talk, for the present and future way of how to support mathematical learning processes in school and pre-service teachers' education.

Plenary Lectures

Wednesday, 13th March

12.00 – 13.00

Zbigniew Semadeni

University of Euroregional Economy, Józefów-Warsaw, Poland

semadeni@mimuw.edu.pl

Rhythms, gestures, subitizing and learning to count

According to Edyta Gruszczyk-Kolczyńska, the origins of learning to count are traced back to rhythmic structures which are formed in the brain during the prenatal period (stimulated by auditory rhythms of maternal heartbeats), then to infant's gestures: this, this, this,… and to nursery rhythms and rhythmic counting. This is the first, crucial period; it is followed by a period of mastering counting procedures and the passage to the number concept and arithmetic operations.

Subitizing is the ability of perceiving the number of a small group of items at a glance without counting and immediately knowing how many items are seen. Numerical judgments made for 1 to 4 objects by means of a direct perceptual-apprehension mechanism are very fast, accurate and confident. This ability has a strong genetic component and is a set-before in the sense of David Tall. However, subitizing is limited to a few small numbers and does not lead to the general concept of a natural number, which is based on unlimited counting and the idea so it goes on. The question of whether subitizing is justly regarded as prior and inferior to ordinary counting will be discussed.

Plenary Lectures

Wednesday, 13th March

14.30 – 15.30

Władimir Mitiuszew

Pedagogical University of Cracow, Poland

wladimir.mitiuszew@up.krakow.pl

Higher education in computer epoch: how to teach biology, chemistry, economy, engineering, geography, informatics ... and mathematics?

Mathematical Modeling and Computer Simulations spread through modern science and technology. However, this topic does not take a proper place in higher education, especially at the beginning level. There is a gap between the classical mathematical theory learned by students and its applications. Computer implementations are frequently reduced to orders as in army to put a computer button to get a result. The goal of this talk is an introduction into interdisciplinary approaches to outline Mathematical Modeling using simple mathematical descriptions, making it accessible for first- and second-year students. The main methods and principal schemes are selected and clearly presented as a unified short course Introduction to Mathematical Modeling and Computer Simulations, see the textbook:
www.amazon.com/Vladimir-V.-Mityushev/e/B001K8D332

Plenary Lectures

Friday, 15th March

14.30 – 15.30

Jakub Jernajczyk

Academy of Art and Design, Wrocław, Poland

jjernajczyk@gmail.com

Thinking in images – mathematical inspirations in contemporary conceptual art

Conceptual art, developed from the second part of the twentieth century, emphasizes the ideas which are the basis of the process of creation rather than focusing on the material results of this process. On the one hand this extreme simplification or rejection of the physical representation of the artwork is one of the main criticisms of conceptual art, but at the same time it is a catalyst for pure abstraction and theoretical growth. It is not surprising that many members of this movement, whilst searching for inspiration for their creation, glean from topics of philosophy or mathematics.

During my speech I will present a selection of artworks inspired by concepts of mathematics or philosophy of mathematics, which were created by pioneers of Polish conceptualism as well as by younger artists: M. Aleksandrowicz, J. Chwałczyk, S. Dróżdż, W. Gołkowska, W. Gołuch, Ł. Huculak, J. Jernajczyk, M. Jędrzejewski, E. Smoliński. All of this creators are connected with Wroclaw artistic scene – one of the most important centers of growth of Polish conceptual art. Among these examples one can find artistic realizations in which geometrical analysis becomes a point of entry for philosophical speculation.

Workshop

Thursday, 14th March

17.00 – 18.30, room 118

Piotr Błaszczyk

Pedagogical University of Cracow

piotr.blaszczyk@up.krakow.pl

On Euler's formula. Between standard and non-standard analysis

The workshop combines an interpretation of source texts, specifically (Euler 1755, ch. 3) and (Euler 1748, §§132-134), with methodological investigations on the role of definition in mathematics. The basic truths of modern mathematics, like the series expansions of e^x, $\sin x$,$\cos x$, are definitions of complex analysis, while in the 18th century they were proved as theorems. Strangely enough, for mathematical definitions are considered to be mere conventions, whereas theorems are to be qualified as true or false.

It is well known that it was Euler who introduced the identity$e^{ix} = \cos x + i \sin x$. In his 1748 *Introductio in analysin infinitorum*, he expanded e^x, $\sin x$ and $\cos x$ into series, and through some operations on these series he derived his famous formula. In his development, Euler does not apply any technique of real analysis, not to mention the notion of limit, but rather infinitesimal and infinitely large numbers instead.

The workshop is dedicated to providing an interpretation of Euler's proof in modern nonstandard analysis. Of course, it is out of question that Euler did not know the ultrapower construction that we apply, or any other technique of modern mathematical logic. He also did not know the 19th century real numbers, as well as the 20th century concept of an ordered field. However, we can show that he implicitly applied rules of non-Archimedean fields, and his operations with infinite sums can be interpreted in terms of hyper infinite sums as defined in nonstandard analysis. As an easy introduction into a technique of algebraic interpretation of historical texts, during the Workshop, we will provide an interpretation of Euclid's Elements book V and Descartes' arithmetic of line segments as introduced via diagrams in his 1637 *La Géométrie*.

References

Euler L. (1748), *Introductio in analysin infinitorum*, Lausannae 1748; *Introduction to analysis of the infinite*, transl. by J. Blanton, NY 1988.

Euler L. (1755), *Institutiones Calculi Differentialis*, Petropolitanae 1755; *Foundations of Differential Calculus*, transl. by J. Blanton, NY 2000.

Presentations in sessions

Wednesday

15.30 – 16.30 Session 1A: New technologies and mathematics

Felicitas Pielsticker, University of Siegen, Germany

pielsticker@mathematik.uni-siegen.de

Concept Building in Mathematics Classrooms using New Media: Theoretical Perspectives and Empirical Insights on the Example of 3D-Printing Technology

The demand for integrating so-called new media in schools is, due to political and job market reasons, quite high. STEM educators around the world feel the pressure and the need to design sense-making materials for schools and to integrate new tools into classrooms. In my talk I will present results of my dissertation project regarding this observation. I will first introduce the constructivist concept of "empirical mathematics" relying on physical objects in the sense of Alan Schoenfeld, David Tall, Horst Struve and others. The prevailing idea is that the use of new media may foster the constitution of a so-called empirical belief system – as I will argue in the talk. This is a desirable effect – if we acknowledge that the consequent use of new media changes the way mathematics is taught and understood in classrooms. In the second part of my talk I will report initial results of a study which was conducted over one academic year in a Grade 8 classroom (in a German middle school) to gain insight about how the consequent use of new media – in this case, 3D-printing technology – affects concept-building processes of students.

Sylwia Kania, University of Silesia, Katowice, Poland

sylwia.kania@us.edu.pl

Transgressions between "old" methods of problem solving and "new" students

Nowadays, interpersonal skills are mainly reduced to virtual communication. For many years a lot has been said about digitalized reality. Teachers have been complaining about their students' weak ability of reading and understanding what they had been reading. Young people are used to exchanging short text massages, casual information and mental shortcuts. They are addicted to constant emotional signs delivered by mobile transmitters, and thus they cannot focus on a traditional, printed text – long, unchanging and often monotonous. The whole digitalization process has started many years ago and despite its steady escalation, current "young adults" are already the "digital generation" and written, printed text is much less close to them than messages displayed on their smartphones or notebooks. These "young adults" are, among others, teachers that have a duty to equip their pupils with the basics of mathematical knowledge. School education is strictly connected with verbal communication, clear and complete transmission and, as a consequence, with the use of correct and precise mathematical language. It is necessary to care about communication processes during math classes performed by young teachers and also about the forms of information transfer among student teachers.

16.45 – 17.45 Session 1B: New technologies and mathematics

Katarzyna Wadoń-Kasprzak, Witold Pilecki, University of Applied Sciences in Oświęcim, Poland

kwadon@gmail.com

Forming the concept of parameter through the work with computer algebra software

The aim of the study is to investigate whether software can be useful in a process of forming the concept of parameter. The author also would like to show to what extent computer program can help in understanding the different meanings of this concept during continuous contact with this tool in mathematics learning. Students at the age of 18 used different computer programs during a one-year period. They studied the parameter in different roles: placeholder, generalizer, changing quantity and unknown-to-be-found. The results indicate that the integration of mathematics and information technology could contribute to the pupils' understanding of algebra concepts, particularly the concept of parameter.

Janina Duda, Witold Pilecki, University of Applied Sciences in Oświęcim, Poland

janduda1@wp.pl

Mathematical transgressions of gifted students inspired by using information technology

Major challenges of the modern world, which is dominated by information technology, require people who are innovative, creative, and well educated in mathematics. Creative and innovative actions constitute the most specific type of transgressions. When it comes to education, these are (with some exceptions) psychological transgressions. As Kozielecki (2002) wrote, they "do not create values which are new for mankind or nations, yet they are important from the point of view of individuals". The aim of my talk is to present examples of creative mathematical transgressions of gifted students aged 15-16. When solving properly selected problems, these students draw inspiration from graphing calculators or computer programs, which allow them to transgress symbolic borders set by the limits of their "zone of actual development" (Wygotsky, 1971). These tools serve as a "scaffolding" which supports the students' knowledge and skills that are new for them. It seems really accurate to compare each such transgression to a game. It is a spontaneous and open game, without a clear beginning and end. It is not the success that matters in such a game, but the fact that something new is created (Kozielecki, 2002).

Presentations in sessions

Thursday

10.30 – 12.00 Session 2: Mathematics teaching

Adedeji Tella, University of Ibadan, Nigeria

dejtell@gmail.com

Use of self-regulated learning strategies by senior secondary school mathematics students

The study explores different ways by which senior secondary school mathematics students in Oyo state, Nigeria utilized self-regulated learning strategies to problem-solving among themselves. The study used grounded theory in order to explain how these students in senior school (SS) II classes used strategies to learn material in their school mathematics classes. The study sought to understand the self-regulated learning strategies that students used to learn information, the frequency of their strategies use, and the students' methods of acquiring these strategies. Findings and assertions were drawn from multiple data sources, among others: interviews, focus group discussions, classroom observations and structured questionnaires collected throughout the duration of the study. The results suggest that the use of self-regulated learning strategies involves students' knowledge of cognitive, metacognitive and motivational strategies and students' motivation to use these strategies in mathematics classroom. Furthermore, they suggest that mathematics teachers need to stress the relevance of self-regulated learning strategies to the students and require more reflective, high-level thinking on problem-solving class work and on mathematics examinations.

Mirosława Sajka, Pedagogical University of Cracow, Poland

miroslawa.sajka@up.krakow.pl

The notion of function as a mathematical study of movement - an excerpt from eye-tracking research

The contribution presents an excerpt from an empirical eye-tracking study on the core connections between understanding the notion of function and movement. The research presents an interesting phenomenon. Apart from disclosing the differences between the responses of experts and non-experts to the same tasks, we could observe that the wrong answers were chosen by almost a hundred of well-educated study participants, including mathematics and physics university and PhD students. During the presentation, the probable reasons behind the phenomena will be presented. One has to show a discipline in thinking and manifest the proper usage of everyday life experience and school knowledge to overcome the obstacles which are likely rooted in the role of motion as a primitive notion. This conclusion stands as a starting point to further research.

Roberto Tortora, Università degli Studi di Napoli Federico II, Italy

rtortora@unina.it

Very often students' errors are all but mistakes

The notion of error is certainly crucial in every educational context. Current didactic practices usually face students' errors as they were all mistakes. Therefore strategies are devised and adopted in order to hide or to eliminate the errors or even the errors are just used as convenient tools to assess students' proficiency. But, according to Borasi (1996) [Reconceiving mathematics instruction: a focus on errors. Norwood, NJ: Ablex Publ. Corp.] and to many other authors, errors can be rather conceived as precious didactic tools. In this paper, pointing on these ideas, I try to move a step forward, that is to make an attempt to carefully distinguish several conceptions of errors in educational contexts. The most original and transgressive of such ways of conceiving errors is to reservelessly accept them as partial achievements in a path toward a sharper and sharper accuracy. A number of examples are used to show how this is exactly what happened several times in the history of mathematics and how, from an educational point of view, some difficult notions can be better grasped along this process. Moreover it can be revealed that what is commonly judged an error sometimes can hide a genuine rational behavior.

12.15 – 13.45 Session 3: Mathematics teachers' education

Stanislav Lukáč, Tadeáš Gavala, Pavol Jozef Šafárik University in Košice, Slovakia

stanislav.lukac@upjs.sk, tadeas.gavala@student.upjs.sk

Development of pre-service mathematics teachers' competencies for inquiry-based teaching

Modern conceptions of mathematical education require the application of teaching methods stimulating active acquisition of knowledge and skills. Inquiry-based learning has an important position among innovative approaches to teaching mathematics and science. Successful integration of inquiry-based approaches to teaching mathematics at elementary and secondary schools requires the development of teachers' competencies for the preparation and implementation of inquiry based learning activities. In the paper, an inquiry based activity encouraging the investigation and analysis of probability and causal patterns in games with cards is described. The proposed learning activity was tested on two groups of future mathematics teachers during the course of didactics of mathematics. Sequence of tasks in the form of a worksheet allowed the control of inquiry work and the evaluation of the gained experience. The rules of two card games were described in the first part of the worksheet. The tasks in this part were focused on investigation of the process and results of the card games and listing the discovered findings together with the justification of the answers. The second part of the worksheet included tasks that could lead future teachers to the suggestions for completing teaching materials and preparing inquiry activity for mathematics teaching at secondary school. Various forms of the formative assessment were used in the inquiry activity in order to encourage future teachers to answer questions from the learners' position. Based on experimentation with cards and discovered findings, future teachers had been preparing procedures for students' inquiry and auxiliary questions for conducting discussion with students.

Nelleke den Braber, NHL Stenden University of Applied Sciences, Netherlands

braber@nhl.nl

Jenneke Krüger, Freudenthal Institute, University of Utrecht, Netherlands

jenneke.kruger@gmail.com

Gaining perspective: lessons from interviews with non-mathematics teachers by pre-service mathematics teachers

As part of their teacher training pre-service mathematics teachers are asked to cross boundaries between the mathematics they teach and other school courses in secondary education. Through a compulsory semi-structured interview the pre-service teachers have to research how teachers of other courses deal with the mathematics in their own course. For instance, they find out how mathematical concepts are introduced and taught within other courses and how teachers solve contextual mathematical problems. The pre-service teachers have to construct their own substantiated interview with questions and problems to show that they can make connections between the mathematics they teach and other school courses and disciplines for the benefit of their own students. The assignment is not only meant to increase the teachers' knowledge to effectively teach mathematics but also to put the teaching of mathematics in perspective. We want teachers to look at the relevance of the mathematics they teach and to realise that many of their own students are not aware of the connections between different school courses, for instance, when a similar concept is looked at from different disciplinary perspectives. We argue that this type of knowledge should be a part of mathematical knowledge for teaching as described by Ball, Thames & Phelps (2008). In this presentation we will show what choices the teachers made in their interviews, how they reflected on their own teaching and why we believe this knowledge is important for participating in interdisciplinary courses within secondary education.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, 59(5), 389-407

Basia Pieronkiewicz, Pedagogical University of Cracow, Poland

barbara.pieronkiewicz@up.krakow.pl

High-school mathematics textbooks as the potential source of pre-service teachers' images of a line tangent to a plane curve

Research on textbooks have been an important part of mathematics education research for many years. The initial approach towards the textbooks was based on the assumption that they strongly influence school students' learning. However, as it has been already shown by many studies, mathematics textbooks also play an important role in pre-service teachers' vocational preparation and affect the way novice teachers introduce and develop mathematical concepts in the classroom.

Motivated by the difficulties that, as I observed, many student teachers have with some mathematical concepts, I have analysed a few series of high-school mathematics textbooks used in Poland in order to find out what images of a particular concept of a line tangent to a plane curve they build up to.

In my talk I will try to answer the following questions:

1. It is well known that the paradigmatic model of a line tangent to a circle contributes greatly to the development of certain tangent-related misconceptions. How do the Polish textbooks address this problem? What do the textbooks' authors do in order to prevent students from developing invalid images of a line tangent to a curve and when in the span of high-school mathematics education do they offer an adequate discussion?

2. If a line tangent to a plane curve is not, as many students think, a line having exactly one common point with that curve, then what is it? What could it be in the eyes of the beholders – the recipients of high-school mathematics textbooks? In other words: what is the essential nature of a line tangent to a curve appearing in the textbooks?

15.00 – 16.30 Session 4: History of mathematics

Janet Barnett, Colorado State University – Pueblo, USA

janet.barnett@csupueblo.edu

Transgressions in Nineteenth-century Mathematics: Meaningful Context for Today's Classroom

As an act that "steps across" a law, a code of conduct, or some given bound or limit, a transgression is inherently context-dependent. Not only do laws and social norms vary between different communities, but those adopted by a given community may also change over time. In some instances, a change in a community's accepted codes arises as the result of transgressions committed by its members. Those individual transgressions in turn may be committed unintentionally, or as deliberate acts of violation. In either case, it is possible for a community's expectations to change so drastically that acting in accordance with its recently-held rules will be viewed as a major transgression by at least some of its members.

In this talk, I explore examples of such changes within the history of nineteenth-century mathematics, an era in which professional mathematicians transgressed all previous limits of abstraction, rigor and formality. I illustrate these examples with excerpts from primary historical sources in which leading mathematicians of that time period described the concerns, questions and problems that motivated their work. I also briefly describe how the design of classroom projects, based on these same primary sources, may provide context and meaning for the mathematical learning of today's students by allowing them to witness first-hand the mathematical transgressions of their historical predecessors.

András Benedek, Research Centre for the Humanities, Hungarian Academy of Sciences, Hungary

agbenedek@gmail.com

On the Role of Model Theory in Understanding Mathematical Transgressions: Some examples from the history of continuity, computability and categoricity

Either we think about mathematical transgressions in terms of Zbigniew Semadeni's triad, "deep, surface, formal" or in terms of David Tall's three levels of development, "Practical, Theoretical and Formal", the point has to be driven home: the emergence of mathematical meaning and its communication are intertwined with human activities, among others, producing externalizations and formalizations. The point implies that our understanding of externalizations and formalizations, including the interpretation of these terms, may change both as the result of historical and cognitive development. The interpretative drive of instrumentally augmented human understanding is bootstrapping transgressable limits inherent in any practice. I argue that Modern Model Theory has not only changed the deductive axiomatic meaning of formalisation, but also became a useful tool in reconstructing what went, or goes, on in a particular historical or cognitive process. Formalizations may, however, produce borders of understanding as well. I give examples from the history of uniform continuity, computational models, and problems of categoricity, illustrating that we need a diversified conception of formalization and externalization to understand transgressions. Instead of trying to grasp the elusive, the role of embodied experience in meaning-creating human activity is contrasted with computational models making intersubjective "what is expressed". Theoretical insights from Catarina Dutilh Novaes, Erik Palmgren, and John T. Baldwin assist to reconstruct the meanings of "formal" and "external" in the contexts of the examples underlining the importance of the choice of logic, vocabulary, and the point of reverse mathematics in interpreting the "same" propositions in different systems.

Marlena Fila, Pedagogical University of Cracow, Poland

marlena.fila@up.krakow.pl

On continuity in Bolzano's 1817 *Rein analytischer Beweis…*

Ancient Greek characteristic of continuity was equally applied to space, time and motion, and was encapsulated in the statement: continuous thing is divisible into parts that are infinitely divisible (see [1]). In modern science, space and time are represented by real numbers, assuming R^n models space and $(R, <)$ models time, while motion is represented by function. Accordingly, continuity splits into continuous order (continuity of real numbers) and continuous function. I argue that this split of the meaning of continuity started in [2] and resulted from a duality of geometric line and function.

Marked by that duality of line and function, [2] belongs to the tradition (initiated by [2], developed in [4]) under which an analytic formula represents function, diagram represents line, and the relation between function and line is guaranteed by some non-mathematical conditions. That duality was covered by the arithmetization of analysis, and then completed in the set-theoretic foundations of mathematics. Under the set-theoretic definition of function, a line is the graph of a function; when line is identified in R^n its continuity is related to least upper bound principle. On the other hand, the continuity of function is related to definition of the continuous function and echoes its "mechanical" provenance.

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Presentations in sessions

Friday

10.30 – 13.00 Session 5: Communication and mathematics

Jorge Soto-Andrade, University of Chile, Chile

sotoandrade@uchile.cl

Daniela Diaz-Rojas, University of Oxford, UK

Alexandra Yañez-Aburto, University of Chile, Chile

alexandra.yanez@ug.uchile.cl

Metaphorising and enacting as a means of transgression in the learning of mathematics

We claim that metaphorising and enacting are fundamental means of transgression in the learning of mathematics. Indeed, when a problem is "given" to a learner, the latter remains usually confined within the terms of a didactic contract (in the sense of Brousseau), which sets stringent boundary conditions for his or her acting and reacting. The fact of disliking the problem (a legitimate affective reaction) may trigger metaphorising from the learner, which often changes the problem in a dramatic way. This is often done through idiosyncratic metaphorising, a conspicuous avatar of transgression. Indeed, metaphorising involves border crossings of all sorts, not only within mathematics itself but also between different realms in science and human culture. Also an enactivist approach to the learning of mathematics is often quite transgressive: we just pose or suggest a "situational seed" to the learners and keep silent, so that students have room to ask questions on their own: an unfamiliar and transgressive situation. So an enactivist approach allows for the emergence of what is unlikely and new, "out of the box" regarding the confining didactic contract. This brings forth amazement and surprise, key ingredients for learning (cf. Peirce). We report here on our transgressive metaphoric and enactivist experimenting with various cohorts of learners in Chile these last years including: first year university students majoring in social sciences and humanities, prospective mathematics and physics teachers, prospective mathematicians, in service primary school teachers engaged in a professional development programme, primary and secondary school students in pilot experiments.

Gaya Jayakody, University of Sri Jayewardenepura, Sri Lanka

gayanayomi@sjp.ac.lk

Realizations and commognitive conflicts in the discourse of 'continuous functions'

This study reports on some of the realizations and commognitive conflicts around the discourse of continuous functions among university first year students taking Calculus courses. The study adopts the commognitive theory of learning as opposed to different cognitive theories that have been used in previous studies on learning continuous functions. Students' communication about continuous functions and the process of discourse along with its patterns and mechanisms is observed through the commognitive lense. The study focused on two definitions for continuous functions (and for continuity at a point) from the literature, which in some cases lead to inconsistent conclusions with respect to the continuity of a given function. In this study, I identify a number of realizations of continuous functions (different ways in which students communicate about a continuous function) and discuss how some of these realizations are challenged when functions with different domains are presented.

Janet Barnett, Colorado State University – Pueblo, USA, janet.barnett@csupueblo.edu

Cihan Can, Florida State University, USA

Kathleen Clark, Florida State University, USA, kclark@fsu.edu

Learning Mathematics from Primary Sources:
Meta - Discursive Rules, Exogenous Growth and Transgressive Acts

Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) is a five-year project, funded by the US National Science Foundation, which seeks to develop, classroom-test and evaluate the effects of curricular materials, based on primary sources, that are designed for use in undergraduate mathematics courses. As the framework for one component of our research, we adopt Sfard's theory of "mathematics as a discourse" and its distinction between object-level rules and meta-discursive rules (Sfard 2008). Within this framework, expansions of a discourse at the object-level are generally associated with "endogenous growth," while those involving changes in meta-discursive rules are indicative of "exogenous growth." Due to certain characteristics of meta-discursive rules (e.g., value-ladedness), exogenous growth is considered especially difficult to achieve.

Through intensive fieldwork spanning three semesters, our meta-discursive rules investigation has sought to examine the effects of exposing students, through their engagement with TRIUMPHS Primary Source Projects, to mathematical discourses that are governed by different meta-discursive rules. The use of a pedagogical approach that embraces this type of historical perspective further allows us a lens to explore mathematical discourse at the macro-, meso- and micro-levels (Vaughan 2002). In this talk, we relate our analysis of data to these three levels as a means to explore connections between meta-discursive rules and the concept of mathematical transgressions. We illustrate, for instance, how adoption of a new meta-discursive rule by an individual represents a transgressive act at the micro-level. We also consider how transgressive acts may serve as a precursor to exogenous growth.

Ioannis Vandoulakis, Hellenic Open University, Greece

i.vandoulakis@gmail.com

Proof-Events as a Fundamental Methodological Concept
Integrating History of Mathematics into Intercultural Mathematics Education

Much effort and energy has been spent during the last decades to integrate elements of history of mathematics, history of science and culture into mathematics curricula in various levels of formal education. This has raised many debates between mathematicians, historians of mathematics and mathematics educators.

The mathematical proofs that historians face in the extant sources involve many informal components, often not completely formalizable, and convey some kind of semantic content calling for understanding and verification. These materials can hardly be used straightforwardly in the classroom by mathematics educators, because of their stylistic and communicative characteristics that often prevent understanding by a contemporary reader.

The concept of proof-event or proving (instead of proof) which was introduced by Joseph Goguen, can serve as fundamental methodological tool for integrating history of mathematics into mathematics education, taking into account the cultural and social aspects of mathematics.

Proof-events are understood as spatio-temporal social processes that involve at least two agents enacting two distinct roles: a prover and an interpreter. Thus, proving is viewed as cooperative problem-solving activity of a multi-agent system evolving in space and time that generates proofs articulated in various semiotic codes and communicated in different styles (cultural contexts). Insofar as proof-events take place in time, they are themselves, as well as the semiotic and communication codes used for conveyance of information, embedded in history; thus, they also include their own proving history.

These aspects of proof-events, that evolve in real (historical-social) space, or in a virtual (networked-social) research or education space, in a sense similar to the case of the Polymath project, can serve as a model for organizing similar collaborative discovery proof-events in mathematics education practice, in particular in collaborative problem-solving learning situations in the real classroom or in a virtual class.

Karl Heuer, Technical University of Berlin, Germany

kmh@exmpl.de

Deniz Sarikaya, Universität Hamburg, Germany

Deniz_Sarikaya@hotmail.de

Variations in open problem fields as a tool for mathematical education: From basics to open questions in the twinkling of an eye

Mathematics is a creative endeavour, especially on a research level. We want to communicate this openness to pupils and students by integrating it into their mathematical education. First, we discuss how simple syntactical variations of statements can lead to new propositions to study. We shall show how far this mechanism can be used in mathematical education to develop a more open, i.e. research oriented, experience for participating students. Making use of this, we explain how teachers could work within such an open research-oriented framework in general. Maybe against initial expectations, this does not necessarily involve an increased effort in preparation. As an example, we present five designed worksheets, which can be used consecutively, but do not have to. The purpose of these worksheets is to show the participating students how it feels to 'navigate in unknown territory'. As superordinate subject we consider 'tilings'. We begin with the basic question which regular (convex) polygon can tile the plane. Small variations of this question quickly lead to new sensible fields of study or even to the edge of research. For example, allowing to combine different regular (convex) polygons leads to Archimedean plane tilings, or introducing the notion of 'periodicity' paves the way for questions related to Penrose tilings. This talk is based on work used for enrichment programs for mathematically gifted children and on observations from working mathematicians.

15.45 – 17.15 Session 6: Mathematics and Arts

Melissa Rodd, UCL Institute of Education, United Kingdom

m.rodd@ucl.ac.uk

Introducing mathematics through art, a case study: Maryam Mirzakhani's geometry for high school students

Maryam Mirzakhani (1977-2017) was awarded the Fields Medal in 2014 for her work about curves on Riemann surfaces (surfaces described by complex numbers). As part of Mirzakhani's personal working method she drew diagrams which had an aesthetic appeal and provided her opportunities for insight without (at that time) doing detailed calculations. Although it would take many years of study in geometry, dynamical systems and analysis to understand the mathematics she produced at a professional level, as teachers of mathematics we may consider whether some access to this mathematical world might be possible via aesthetic experience. Materials for 16-19 year olds who are studying specialist mathematics courses at high school will be presented. These materials introduce notions of curvature, geodesics and topology through aesthetic, visual or tactile experience. The purpose of designing such materials is threefold: (1) Participation in STEM: introducing 16-19 year olds to the first female and first Iranian Fields Medal winner, providing information about features of Mirzakhani's career that she attributed to facilitating her participation in mathematics at the highest level; (2) Art experience: using easily found equipment, models and drawings can be made by students and may give insight into notions of curvature, geodesics or topology; (3) Aesthetics of mathematics: paintings, sculptures or instillations may afford a feeling of transcendence, insight or delight. There are many issues to discuss related to this presentation including tensions between: mathematical precision and 'having a feel for' mathematics; affective pleasure vs. uncomfortable dogged determination; visual vs. analytic proofs.

Agata Hoffmann, University of Wrocław, Poland

agata.hoffmann@math.uni.wroc.pl

Zvi Hecker and his mathematical inspirations

In the process of teaching-learning geometry we usually have a lot of problems. Although we live in a three-dimensional world and we are accustomed with two-dimensional surfaces, to see it through the eyes of mathematics it is not an easy task. In my presentation, I would like to show chosen works of Zvi Hecker, which I think can be an inspiration for teachers, while they approach some of mathematical topics.

In the world around us it is not difficult to find examples of cuboids. With prisms it is slightly more difficult, but we can manage. With other shapes - polyhedrons it is almost impossible. So, when I saw the examples of Zvi Hecker`s works I was amazed.

Alfred Neumann published in 1956 his article *L`Humanisation de l`Espace: Le Systeme M-Φ* (The Humanisation of Space: The System M-Φ) in which he takes M (metre – a fundamental unit of longitude measure) and Φ (phi - a mathematical constant that describes the Golden Ratio) and produces a series of numerical grids that relate to one another. According to the author by using this relations we "humanize" the space. Based on this idea Zvi Hecker and his colleague Eldar Sharon and their mentor Alfred Neumann constructed City Hall in Bat Yam (Israel). Using this project and three others – Club Mediterrane in Achziv (Israel), the Synagogue of Military Academy Campus at Negeve Desert (Israel) and Areospace Engineer Lab Technion in Haifa (Israel), I would like to share a few examples of the way of approaching geometry items at schools.

The next mathematical issue which we can see in Zvi Hecker`s works is the logarithmic spiral. The architect discovered the beauty of this spiral in the sunflower head and used this idea in his works. To show the way of transforming and using this mathematical idea I will concentrate on four projects: Dubnier House in Ramat Gan (Israel), City Centre of Ramat ha- Sharon (Israel – urban project), Haintz-Galinski Jewish School in Berlin (Germany) and Ramote Housing I in Jerusalem (Israel). This situation in which mathematics is the source of creativity in a field not strictly connected to it might appeal to some of our pupils and make them see that it is worthwhile to learn mathematics.

Michaela Kaslová, Charles University, Czech Republic

michaela.kaslova@pedf.cuni.cz

Paola Vighi, Università di Parma, Italy

How young pupils perceive geometrical shapes and structure?

This study focuses on the investigation of the ability to see, inspect and observe geometrical objects, starting from an abstract art painting, in pupils 5-6 years old. The first approach to the geometry is based on spatial experiences, observation of the reality and visualisation. In kindergarten, it can be useful to propose the manipulation of shapes, to improve children knowledge and to promote the transition from static to dynamic images. A qualitative research was implemented in the interdisciplinary field “Art and Mathematics”. The analysis of young children behaviours and difficulties suggests some interesting from the mathematical point of view results and new ideas of how to improve the classroom activity.

Presentations in sessions

Saturday

10.30 – 12.30 Session 7: Affect and mathematics

Bronisław Czarnocha, Hostos CC, CUNY, NYC, USA

Gerald Goldin, Rutgers University, USA

Peter Liljedahl, Simon Fraser University, Canada

Illuminating Aha!Moment through the Analysis of Relationships between Affect, Cognition and Conation

The presentation addresses the nature of Aha!Moment, which in recent times became the focus of the new approach to creativity in mathematics classroom (Czarnocha et al, 2016). The presentation illuminates Aha!Moment through the analysis of relationships between affect, cognition and conation. Here, affect refers to the emotional interpretation of perceptions, information, or knowledge; cognition refers to the process of coming to know and understand while conation refers to the connection of knowledge and affect to behavior and is associated with the issue of "why." The affective and cognitive presence during the Eureka experience dates to its first manifestation by Archimedes whose affect sent him, after he discovered the buoyancy law, to jump out of the washtub-'his laboratory' and run naked through Syracuse screaming 'Eureka'. However, the role of conation in that phenomenon is less clear. According to Goldin (2019) conation allows us to deeply address the question of why what the student is doing matters to him or her. It will help us answer the question why students, learners engage into the path leading to Aha!Moment insight, what type of motivation drives them at the very moment of insight, what are the ways "in which productive mathematical engagement relates to student needs, desires and will…." (p.5).

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Kathleen Clark, Florida State University, USA

kclark@fsu.edu

Cihan Can, Florida State University, USA

Primary Source Projects as Transgressive Acts: Perspectives on Affective Experiences

The Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) project is a five-year (2015-2020) effort funded by the US National Science Foundation, and seeks to develop, classroom-test, and evaluate the effects of curricular materials, based on primary sources, that are designed for use in undergraduate mathematics courses. We call these curricular materials "primary source projects" or PSPs.

We have collected a variety of data over the last three years to capture dimensions of students' experience with PSPs. For instance, we survey students upon completion of a PSP with a series of Likert and open-ended items. Two of the open-ended survey items ask students to identify benefits and drawbacks of learning mathematics by reading primary historical sources in a given PSP. Students' responses to these items are both helpful and intriguing. On the one hand, we gain valuable information regarding students' experiences with PSPs, that can be analyzed in terms of a variety of factors (e.g., gender, race, year at university, major discipline), and then shared with instructors and PSP authors. On the other hand, we can examine student reflections to identify transgressive actions (Kozielecki, 1987, 1997, as cited in Pieronkiewicz, 2015) that PSPs promote. In our analysis thus far, we have especially noted transgressive actions that "are oriented toward a meaningful change" of students' experience with primary sources, are expressive of components that are inner-directed (e.g., creativity, knowledge, motivation, and perseverance), and are "accompanied by positive affective experiences" (Pieronkiewicz, p. 1262).

Monika Szczygieł, Pedagogical University of Cracow, Poland

Monika.szczygiel@up.krakow.pl

Math anxiety of mothers, fathers, and teachers explains math anxiety and mathematical performance of early school-age children

The study investigated the relationship between math anxiety of parents and teachers and math anxiety and mathematical performance of early school-aged children. The results show that the math anxiety of fathers (but not mothers and teachers) is associated with math anxiety of the first-grade children and third-grade girls. Math anxiety of mothers and teachers (but not fathers) explains the level of mathematical achievement of third-grade students. The research results point out to the importance of adults in shaping pupils' math anxiety and mathematical achievement, but these relationships vary depending on the class grade.

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List of Participants

Janet	**Barnett**	janet.barnett@csupueblo.edu	Colorado State University - Pueblo	United States
András	**Benedek**	agbenedek@gmail.com	Research Centre for the Humanities, Hungarian Academy of Sciences	Hungary
Piotr	**Błaszczyk**	piotr.blaszczyk@up.krakow.pl	Pedagogical University of Cracow	Poland
Nikola	**Budnicka**	nikola.b@wp.pl	Szkoła Podstawowa nr 47 in Szczecin	Poland
Kathleen	**Clark**	kclark@fsu.edu	Florida State University	United States
Bronislaw	**Czarnocha**	bronisuavec2@gmail.com	Hostos CC, CUNY, NYC, USA	United States
Nelleke	**den Braber**	braber@nhl.nl	NHL Stenden University of Applied Sciences	Netherlands
Janina	**Duda**	janduda1@wp.pl	Witold Pilecki University of Applied Sciences in Oświęcim	Poland
Marlena	**Fila**	marlena.fila@up.krakow.pl	Pedagogical University of Cracow	Poland
Tadeáš	**Gavala**	tadeas.gavala@student.upjs.sk	Pavol Jozef Šafárik University in Košice	Slovakia
Gerald	**Goldin**	geraldgoldin@dimacs.rutgers.edu	Rutgers University	United States
Burian	**Hana**	burian.hana69@gmail.com	Eötvös Loránd Scientific University Budapest	Hungary
Karl	**Heuer**	kmh@exmpl.de	Technical University of Berlin	Germany
Agata	**Hoffmann**	agata.hoffmann@math.uni.wroc.pl	University of Wrocław	Poland
Gaya	**Jayakody**	gayanayomi@sjp.ac.lk	University of Sri Jayewardenepura	Sri Lanka
Jakub	**Jernajczyk**	jjernajczyk@gmail.com	Academy of Art and Design, Wrocław, Poland	Poland
Sylwia	**Kania**	sylwia.kania@us.edu.pl	University of Silesia in Katowice	Poland
Michaela	**Kaslová**	michaela.kaslova@pedf.cuni.cz	Charles University	Czech Republic
Małgorzata	**Korwin**	mkkorwin@gmail.com	I Liceum Ogólnokształcące im. B. Nowodworskiego w Krakowie	Poland
Ireneusz	**Krech**	ikrech@up.krakow.pl	Pedagogical University of Cracow	Poland
Jenneke	**Krüger**	jenneke.kruger@gmail.com	Freudenthal Institute Utrecht University	Netherlands
Elżbieta	**Leszczyńska**	sekretariat.wko@ko.poznan.pl	Kuratorium Oświaty w Poznaniu	Poland
Stanislav	**Lukáč**	stanislav.lukac@upjs.sk	Pavol Jozef Šafárik University in Košice	Slovakia

Marjon	**Minderhoud**	Marjon.minderhoud@nhl.nl	NHL Stenden University of Applied Sciences	Netherlands
Władimir	**Mitiuszew**	wladimir.mitiuszew@up.krakow.pl	Pedagogical University of Cracow	Poland
Felicitas	**Pielsticker**	pielsticker@mathematik.uni-siegen.de	University of Siegen	Germany
Basia	**Pieronkiewicz**	barbara.pieronkiewicz@up.krakow.pl	Pedagogical University of Cracow	Poland
Justyna	**Płonka**	juuustynaplonka@gmail.com	Pedagogical University of Cracow	Poland
Melissa	**Rodd**	m.rodd@ucl.ac.uk	UCL Institute of Education	United Kingdom
Bożena	**Rożek**	bozena.rozek@up.krakow.pl	Pedagogical University of Cracow	Poland
Mirosława	**Sajka**	miroslawa.sajka@up.krakow.pl	Pedagogical University of Cracow	Poland
Deniz	**Sarikaya**	Deniz_Sarikaya@hotmail.de	Universität Hamburg	Germany
Zbigniew	**Semadeni**	semadeni@mimuw.edu.pl	University of Euroregional Economy, Józefów-Warsaw	Poland
Anna	**Sfard**	sfard@netvision.net.il	University of Haifa	Israel
Izabela	**Solarz**	izasolarz@interia.pl	Pedagogical University of Cracow	Poland
Jorge	**Soto-Andrade**	sotoandrade@uchile.cl	University of Chile	Chile
Halina	**Syguda**	hsyguda@vp.pl	Szkoła Podstawowa nr 47 in Szczecin	Poland
Monika	**Szczygieł**	szczygiel89@gmail.com	Pedagogical University of Cracow	Poland
David	**Tall**	davidtall@mac.com	University of Warwick	United Kingdom
Adedeji	**Tella**	dejtell@gmail.com	University of Ibadan	Nigeria
Roberto	**Tortora**	rtortora@unina.it	Università degli Studi di Napoli Federico II	Italy
Elżbieta	**Urbańska**	elzbieta.urbanska@up.krakow.pl	Pedagogical University of Cracow	Poland
Ioannis	**Vandoulakis**	i.vandoulakis@gmail.com	Hellenic Open University	Greece
Katarzyna	**Wadoń-Kasprzak**	kwadon@gmail.com	Witold Pilecki University of Applied Sciences in Oświęcim	Poland
Ingo	**Witzke**	witzke@mathematik.uni-siegen.de	University of Siegen	Germany
Daniel	**Wójcik**	daniel.wojcik@up.krakow.pl	Pedagogical University of Cracow	Poland
Alexandra	**Yañez**	alexandra.yanez@ug.uchile.cl	Universidad de Chile	Chile
Lidia	**Zaręba**	lidia.zareba@up.krakow.pl	Pedagogical University of Cracow	Poland
Rina	**Zazkis**	zazkis@sfu.ca	Simon Fraser University	Canada